Methods for Solving Quadratic Equations

SQUARE ROOT PROPERTY

This method is used if the form of the equation is $x^2 = k$ or $(ax+b)^2 = k$ (where *k* is a constant).

To solve by the square root property:

1. Isolate the perfect square on one side and a constant on the other side.
2. Take the square root of both sides. NOTE: the square root of a constant yields positive and negative values.
3. Solve the resulting equation.

Example: Solve $2(x-3)^2 - 56 = 0$

1. To isolate the square move the constant, 56, to the right side. $2(x-3)^2 = 56$
   Then divide both sides by 2. $\frac{2(x-3)^2}{2} = \frac{56}{2}$
   $(x-3)^2 = 28$
2. Take the square roots of both sides. Use $\pm$ on the right. $\sqrt{(x-3)^2} = \pm\sqrt{28}$
3. Solve the resulting equation. $x - 3 = \pm 2\sqrt{7}$
   $x = 3 \pm 2\sqrt{7}$

COMPLETING THE SQUARE

A method that can solve ALL quadratic equations.

To complete the square of $\boldsymbol{a}\mathrm{x}^2 + \boldsymbol{b}\mathrm{x} + \boldsymbol{c} = 0$:

1. Isolate $\boldsymbol{c}$, the constant, on one side of the equation.
2. The coefficient of the x^2 term needs to be one. If $\boldsymbol{a}$ does not equal 1, then divide each term by $\boldsymbol{a}$.
3. Divide $\boldsymbol{b}$, the *x* coefficient, by 2 (or multiply $\boldsymbol{b}$ by $\frac{1}{2}$) and then square that value.
4. Add the result found in step 3 to both sides of the equation.
5. Factor the quadratic side (which is a perfect square because we just made it that way!).
   NOTE: The equation should look like (x ± number)^2 = NUMBER
6. Use the Square Root Method to solve the equation.

Example: Solve $3\mathrm{x}^2 + 5\mathrm{x} - 6 = 0$

1) Add 6 to both sides of the equation. $3\mathrm{x}^2 + 5\mathrm{x} = 6$

2) The coefficient of the x^2 term must be one. Divide all terms by 3. $\frac{3\mathrm{x}^2}{3} + \frac{5\mathrm{x}}{3} = \frac{6}{3}$

$\mathrm{x}^2 + \frac{5}{3}\mathrm{x} = 2$

3) Multiply $\frac{5}{3}$ by $\frac{1}{2}$ and then square the result. $\frac{1}{2} \cdot \frac{5}{6} = \frac{5}{6} \rightarrow \left(\frac{5}{6}\right)^2 = \frac{25}{36}$

4) Add $\frac{25}{36}$ to both sides of the equation. $\mathrm{x}^2 + \frac{5}{3}\mathrm{x} + \frac{25}{36} = 2 + \frac{25}{36}$

5) Factor the left side and add the fractions on the right side.

Notice: $\frac{5}{6} = \sqrt{(25/36)}$ $\left(x + \frac{5}{6}\right)^2 = \frac{72}{36} + \frac{25}{36}$

6) Now apply the square root property and solve for x. $\sqrt{\left(x + \frac{5}{6}\right)^2} = \pm\sqrt{\frac{97}{36}}$

$\mathrm{x} + \frac{5}{6} = \pm\frac{\sqrt{97}}{\sqrt{36}}$

$\mathrm{x} = -\frac{5}{6} \pm \frac{\sqrt{97}}{6}$ OR $\frac{-5 \pm \sqrt{97}}{6}$

FACTORING

This method only works if the quadratic polynomial can be factored.

To solve by factoring:

1. Write the equation in standard form (all terms on one side and equal to 0).
2. Factor the polynomial.
3. Set each factor equal to zero.
4. Solve the resulting equation(s).

Example: Solve $x^2 + 5x = 24$

1. Put the equation in standard form, subtract 24 from each side. $x^2 + 5x - 24 = 0$
2. Factor the polynomial on the left. $(x + 8)(x - 3) = 0$
3. Set each factor equal to zero. $x + 8 = 0$ or $x - 3 = 0$
4. Solve each linear equation. $x = -8$ or $x = 3$

QUADRATIC FORMULA

A method that can solve ALL quadratic equations.

To solve $\boldsymbol{a}\mathrm{x}^2 + \boldsymbol{b}\mathrm{x} + \boldsymbol{c} = 0$ with the quadratic formula:

1. Write the equation in standard form (all terms on one side and equal to 0).
2. Plug the numbers for $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ into the formula shown below:
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$
3. Use the order of operations to simplify the expression.
   Note: simplify the radical when possible.

Example: Solve $x^2 + 5x = 24$

1. Put the equation in standard form, subtract 24 from each side. $x^2 + 5x - 24 = 0$
2. Plug in 1 for a, 5 for b, and -24 for c into the quadratic formula. $x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-24)}}{2(1)}$
3. Follow the order of operations. Start with the operations inside the radical first. $x = \frac{-5 \pm \sqrt{121}}{2(1)}$
   $x = \frac{-5-11}{2}$ or $x = \frac{-5+11}{2}$
   Note: when the answers are rational, "nice" numbers, then we could have used the factoring method as seen on the previous example. $x = -8$ or $x = 3$

Example: Solve $-4x^2 = -12x + 11$

1. Put the equation in standard form, add $4x^2$ to each side. $0 = 4x^2 - 12x + 11$
2. Plug in 4 for a, -12 for b, and 11 for c into the quadratic formula. $x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(11)}}{2(4)}$
3. Follow the order of operations. Start with the operations inside the radical first.
$$x = \frac{12 \pm \sqrt{-32}}{2(4)}$$
$$x = \frac{12 \pm i\sqrt{32}}{2(4)}$$
$$x = \frac{12 \pm 4i\sqrt{2}}{8}$$
$$x = \frac{12}{8} \pm \frac{4\sqrt{2}}{8}i$$
$$x = \frac{3}{2} \pm \frac{\sqrt{2}}{2}i$$