LOGARITHMS

Basic Ideas of Logs:

1. $\log_b x$ is read as "log with base *b* of *x*"

2. Common log (base 10): $\log_{10} x$ is equivalent to $\log x$

3. Natural log (base *e*): $\log_e x$ is equivalent to $\ln x$

4. Logarithms are the inverses of exponentials (just like subtraction and addition).

5. $\log_b x = y$ ...is the same as... $x = b^y$
   For example: $\log_3 \frac{1}{81} = -4$ ...is the same as... $3^{-4} = \frac{1}{81}$

Properties of Logarithms:

1. $\log_b x + \log_b y \leftrightarrow \log_b xy$ — **Example:** $\log_3 4 + \log_3 6 \leftrightarrow \log_3 4 \cdot 6 = \log_3 24$

2. $\log_b x - \log_b y \leftrightarrow \log_b \frac{x}{y}$ — **Example:** $\log_3 10 - \log_3 5 \leftrightarrow \log_3 \frac{10}{5} = \log_3 2$

3. $r \log_b x \leftrightarrow \log_b x^r$ — **Example:** $2\log_3 4 \leftrightarrow \log_3 4^2 = \log_3 16$

4. $\log_b b^x = x$ — **Examples:**
   - $\ln e = 1$
   - $\ln e^x = x$
   - $\log_4 4^5 = 5$

5. $b^{\log_b x} = x$ — **Examples:**
   - $e^{\ln x} = x$
   - $5^{\log_5 9} = 9$

6. $\log_b 1 = 0$ — **Example:** $\log_7 1 = 0$

Change of Base Formula:

Changes logarithms with other bases to logarithms with a common base, such as base *e* or base 10.

$\log_b x = \frac{\log_B x}{\log_B b}$

Example: $\log_3 4 = \frac{\log_{10} 4}{\log_{10} 3} = 1.2618\ldots$

Example: $\log_3 4 = \frac{\ln 4}{\ln 3} = 1.2618\ldots$

Solving Equations Containing Logarithms or Exponents:

1. An equation that has a **log** on one side and some expression on the other side can be rewritten without the **log**. $\mathbf{\log_b x = y \;\leftrightarrow\; x = b^y}$

   Example: $\log_3(x+4) = 2 \;\leftrightarrow\; x + 4 = 3^2$
   $x + 4 = 9$
   $x = 5$

2. An equation that has a **log** with the same base on both sides can be rewritten without the **log**. $\mathbf{\log_b x = \log_b y \;\leftrightarrow\; x = y}$

   Example: $\log_3(3x+1) = \log_3(x+9) \;\leftrightarrow\; 3x + 1 = x + 9$
   $2x = 8$
   $x = 4$

3. An equation that has just exponential expressions where the bases are the same can be rewritten without the bases. $\mathbf{b^x = b^y \;\leftrightarrow\; x = y}$

   Example: $5^{2x-10} = 5^2 \;\leftrightarrow\; 2x - 10 = 2$
   $2x = 12$
   $x = 6$

4. An equation that contains exponential expressions but has different bases on each side of the equation, then the **log** (or **ln**) of each side of the equation is taken and then simplified.

   Example: $3^{2x-10} = 5 \;\leftrightarrow\; \log 3^{2x-10} = \log 5$ Or use **ln**. $\ln 3^{2x-10} = \ln 5$

   $(2x - 10)\log 3 = \log 5$

   $(2x - 10)\frac{\log 3}{\log 3} = \frac{\log 5}{\log 3}$

   $2x - 10 = \frac{\log 5}{\log 3}$

   $2x = 10 + \frac{\log 5}{\log 3}$

   $\frac{2x}{2} = \frac{10 + \frac{\log 5}{\log 3}}{2}$

   $x = 5 + \frac{\log 5}{2\log 3}$ Use the calculator to get an estimate.

   $x \approx 5.7324$

5. An equation that has two logarithms with equivalent bases on the same side need to be combined into one log using the properties of logarithms.

   Example: $\ln x + \ln x^2 = 3 \;\leftrightarrow\; \ln x \cdot x^2 = 3$

   $\ln x^3 = 3$

   $e^{\ln x^3} = e^3$ Option: Use property of inverses.

   $x^3 = e^3$

   $(x^3)^{\frac{1}{3}} = (e^3)^{\frac{1}{3}}$

   $x = e$